

**Exercise 22**

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 + 1}}$$

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**Solution**

Evaluate the limit using the limit laws after factoring out the highest power of  $x$  in the denominator.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 + 1}} &= \lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 \left(1 + \frac{1}{x^4}\right)}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2}{x^2 \sqrt{1 + \frac{1}{x^4}}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x^4}}} \\ &= \frac{\lim_{x \rightarrow \infty} 1}{\lim_{x \rightarrow \infty} \sqrt{1 + \frac{1}{x^4}}} \\ &= \frac{\lim_{x \rightarrow \infty} 1}{\sqrt{\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^4}\right)}} \\ &= \frac{\lim_{x \rightarrow \infty} 1}{\sqrt{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x^4}}} \\ &= \frac{1}{\sqrt{1 + 0}} \\ &= 1 \end{aligned}$$